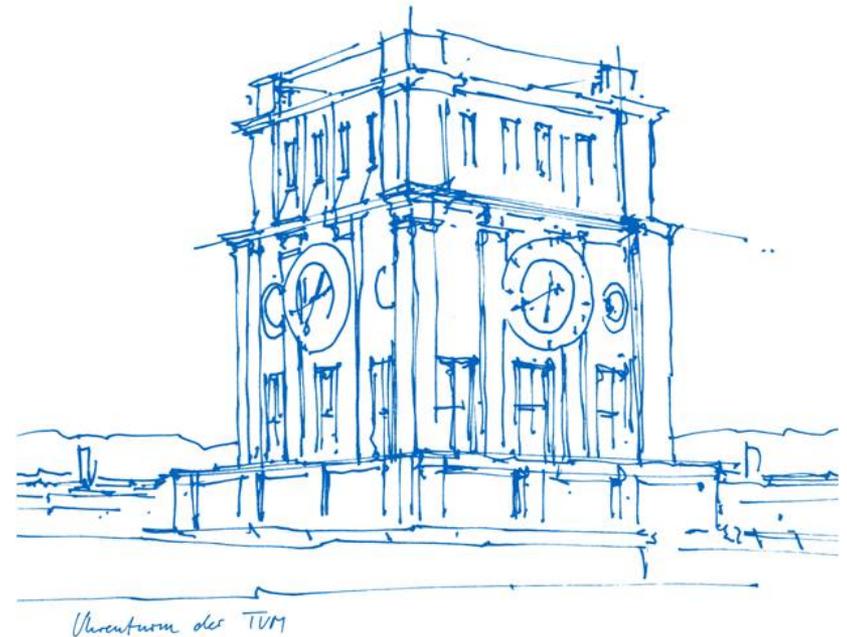
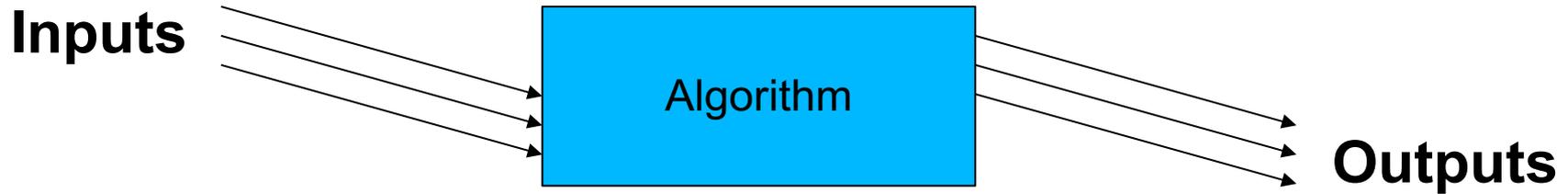


# 07 Cryptography 1

Heiko Niedermayer,  
Georg Carle



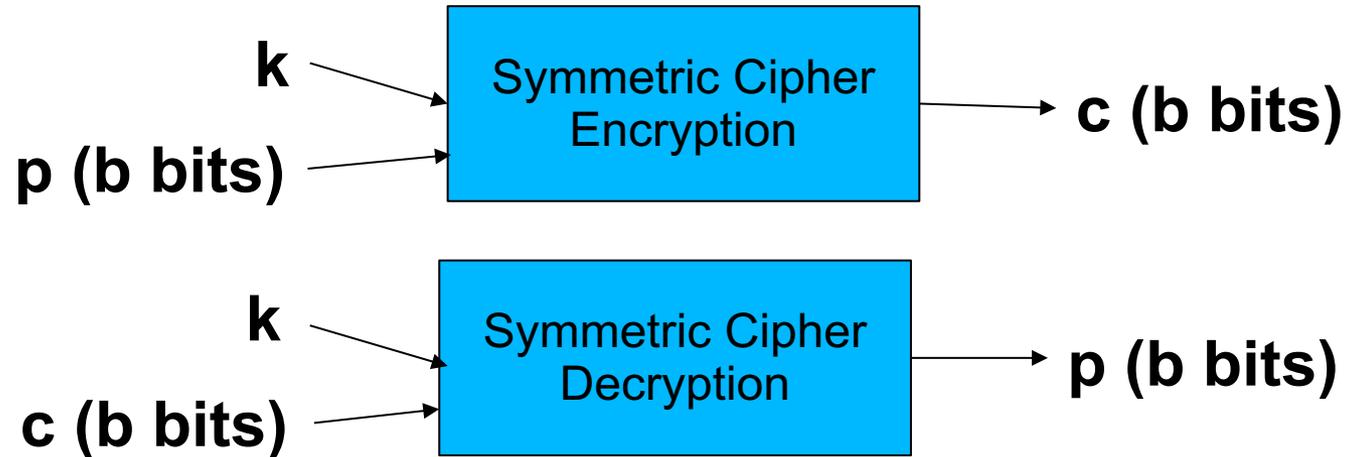
- Overview of Cryptographic Algorithms
- Achieving a Security Goal
- Security Models and Security of Crypto Schemes
- Eavesdropping Experiment
- Chosen-Plaintext Attack



Cryptography provides a variety of algorithms that it uses as building blocks for generating schemes and protocols that achieve a given set of security goals.

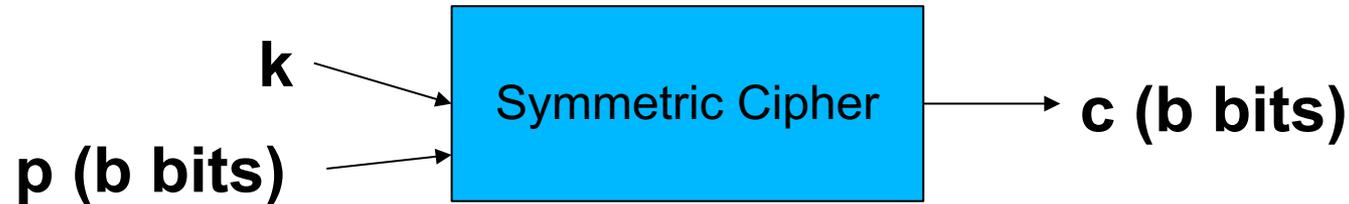
- Symmetric Block Cipher
- Cryptographic Hash Function
- Asymmetric Cipher (Public Key Cryptography)
- ...

- AES-CTR
  - AES is a symmetric cipher
  - Counter Mode (CTR) is a method to encrypt plaintext with a symmetric cipher to achieve confidentiality
  - Inputs: key  $k$ , plaintext  $p$
  - Outputs: ciphertext  $c$
  - Requirements:  $p$  and  $k$  need to remain confidential
- Key Derivation Functions (KDFs)
  - Problem: you have  $k$  bits of entropy / key, needed  $m$  bits for keys
  - Input:  $k_1$  with  $k$  bits
  - Output:  $k_2$  with  $m$  bits
  - Requirements: entropy of  $k_2$  not lower than entropy of  $k_1$
  - KDFs can be built from cryptographic hash functions, symmetric ciphers, ...



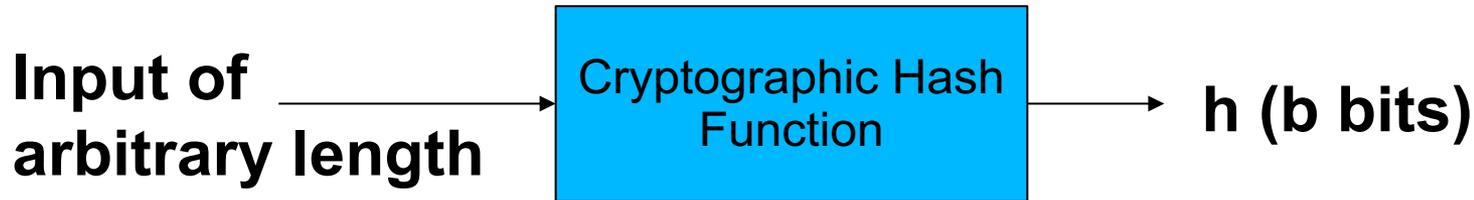
- Block cipher with block length  $b$
- Typical lengths for block and key: 64 bits, 128 bits, 256 bits
- Already known from section on Symmetric Cryptography
  - Note: There are also symmetric stream ciphers. Internally, they may have the concept of blocks as well.

- Operation
  - Needs to contain non-linear element
  - Concept of Confusion
    - Confusion = 0s and 1s can generate output with completely different statistics with respect to 0s and 1s
    - 00000000 can become 11111111, most likely something like 10110100
  - Concept of Diffusion
    - Diffusion = any bit influences bits at other positions, goal: influences all bits
  - Typically, the ciphers repeat a similar set of operations (e.g. for confusion and diffusion) over multiple rounds with round-specific inputs.

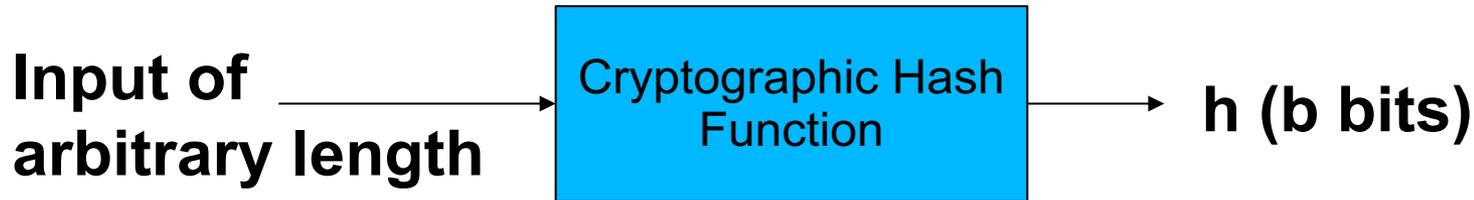


- Evaluation

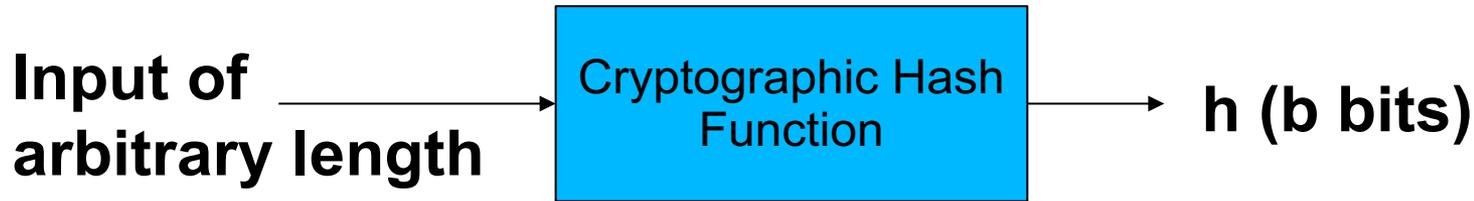
- Application of the block cipher should neither leak the key  $k$  nor the plaintext  $p$ .
- Brute Force attack:
  - Try all possible keys, e.g. given plaintext and ciphertext pair
  - Key  $k$  with  $n$  bits,  $O(2^{n-1})$  average case complexity
- Security of cipher:
  - If best attack on cipher is much better than brute force or if it is computationally feasible, then considered broken.
  - $2^a$  complexity  $\Rightarrow$  a “bits of security”



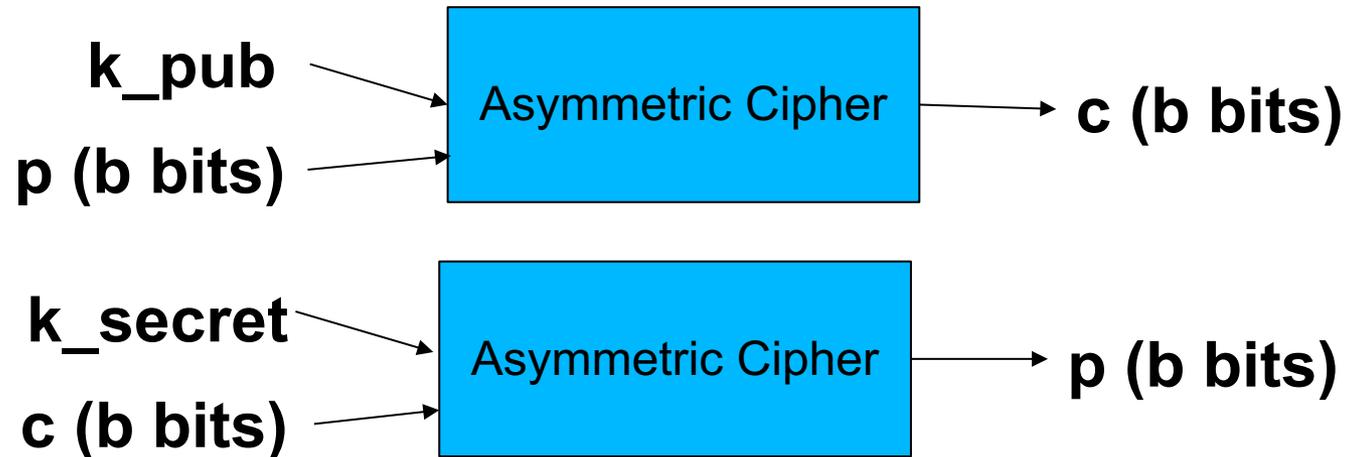
- We will have a chapter on cryptographic hash functions.
- Cryptographic hash functions are hash functions with special properties needed for security, e.g. (not complete list)
  - 1<sup>st</sup> Pre-Image Resistance: Make it hard to find an input  $m$  that produces a given output  $h$
  - Collision Resistance: Make it hard to find two pairs of input  $m_1, m_2$  that produce the same output (collision)



- Operation:
  - Similar to symmetric block ciphers
  - In addition, they need a way to include an arbitrary number of input blocks.
    - e.g. taking the last block of AES-CBC and make key pre-defined would be a cryptographic hash function
    - Traditional hash functions follow Merkle-Damgard constructions
    - Modern hash functions like SHA3 follow other constructions and have finishing functions after processing the last block



- Evaluation:
  - Similar to symmetric block ciphers
  - Due to Birthday Paradox, only half of the bitlength contributes to collision resistance.
    - “160 bit hash function -> 80 bits of security”
    - Collisions can be found for perfect hash function with  $n$  bits output with  $O(2^{n/2})$



- We will have a chapter on asymmetric cryptography with more details.
- Each entity has public  $k_{\text{pub}}$  and secret key  $k_{\text{secret}}$  (also called private key)
- Other entities use the public key  $k_{\text{pub}}_A$  of  $A$  in interactions with  $A$ .  $A$  uses its secret key  $k_{\text{secret}}_A$ .

- Operation
  - Asymmetric ciphers are usually based on mathematical problems that are computationally hard
    - E.g. Factorization (RSA), Discrete Logarithm (ECC, Diffie-Hellman)
    - Most of these problems have efficient quantum algorithms. Thus, these ciphers are not quantum-secure.
- Evaluation
  - Attack the mathematical problem
  - Find weak cases
    - Hard problems are not necessarily hard in all cases. Weak parameters, weak mathematical groups are typical issues faces in asymmetric ciphers.

- Symmetric Cryptography
- Goal: Confidentiality
  
- From the chapter on symmetric cryptography we already know
  - that simply applying the block cipher is not secure (ECB mode)!
  - that CBC or Counter mode provide security.
- How do we know that?
  - Traditionally, schemes were developed and security and insecurity depended on the best attacks found against the method.

- Modern cryptography tries to model the situation of achieving the security goal in a given setting more formally.
- Formal model
  - Needs precise and explicit definition of method and assumptions
  - Allows for mathematical proofs
  - Provides better understanding of properties needed
- Limitation
  - Model  $\neq$  Reality
  - Attacks may still exist, in particular where model assumptions clash with reality.

*Notation:*

$A \leftarrow B$  non-deterministic assignment, can contain some form of randomness

$A := B$  deterministic assignment (no randomness)

$A = B$  comparison

## *Symmetric encryption scheme*

*$k \leftarrow \text{Gen}(1^n)$  # random key is generated and known to the legitimate communication partners*

*$c \leftarrow \text{Enc}_k(m)$ ,  $m \in \{0,1\}^*$*

*$m \leftarrow \text{Dec}_k(c)$*

Such an encryption scheme is considered secure if it succeeds in a theoretical attack game using a chosen-plaintext attack.

In the game, the challenger  $\mathcal{C}$  uses the scheme and adversary  $\mathcal{A}$  tries to overcome the scheme.

# Eavesdropping Experiment

Challenger  $\mathcal{C}$

Adversary  $\mathcal{A}$

$k \leftarrow \text{Gen}(1^n)$

input  $1^n$

$m_0, m_1$

$m_0, m_1$  of  
equal size selected  
by adversary

$b \leftarrow \{0,1\}$

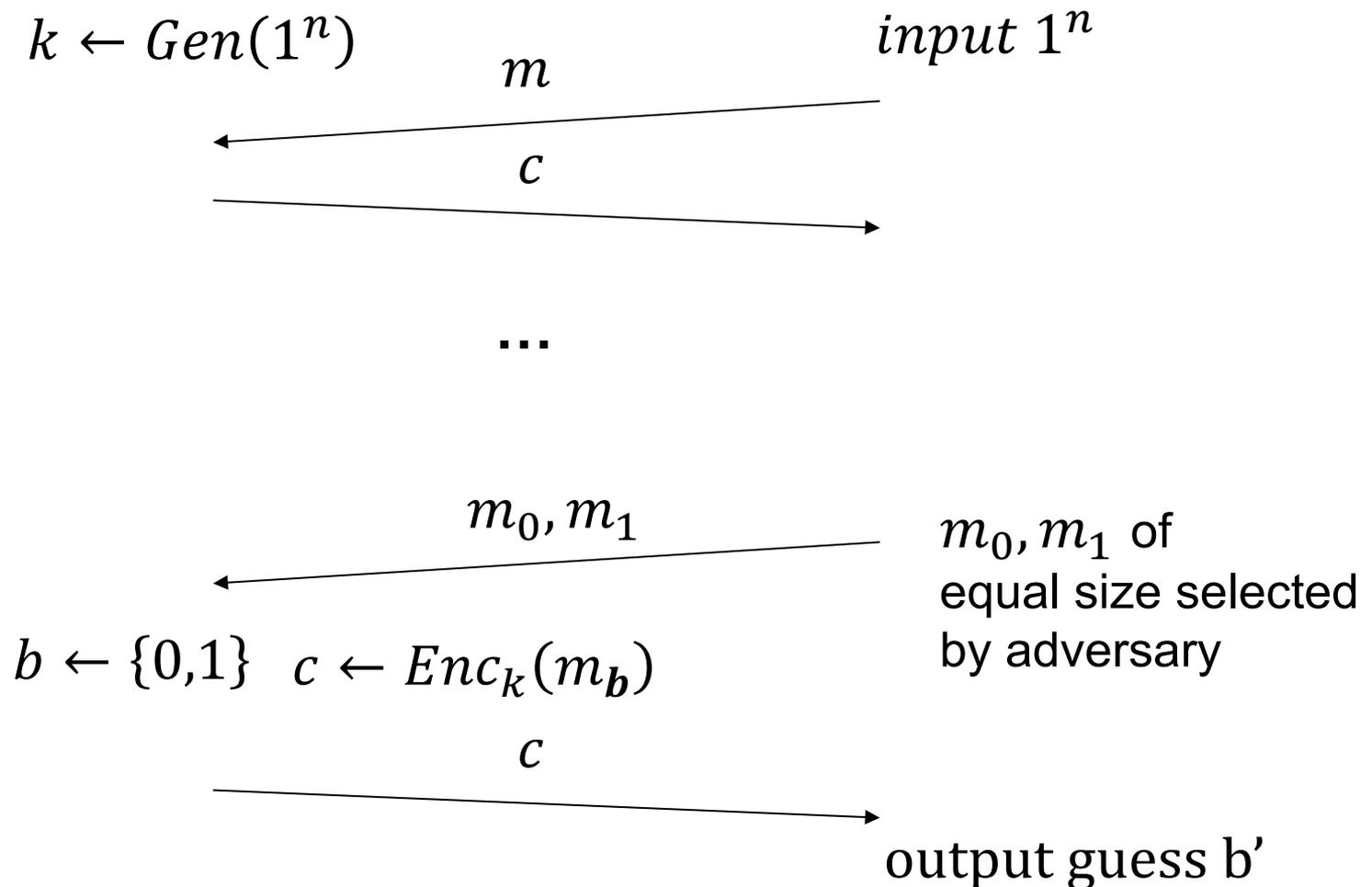
$c \leftarrow \text{Enc}_k(m_b)$

$c$

output guess  $b'$

Adversary  $\mathcal{A}$  succeeds if and only if  $b=b'$

In order to prepare for the game, the adversary is now allowed to utilize information from additional chosen plaintexts. It is allowed a polynomial time chosen plaintext attack.



If the adversary simply guesses, 50 % chance that it will be correct, 50 % that the guess is incorrect. Thus, we cannot expect it to lose the game all the time.

- Enc is secure under Chosen-Plaintext attack (CPA)
  - if this polynomial time-bound adversary is not achieving a success rate above  $0.5 + \text{negligible}$
- This means that the adversary is not able to gain significant information from observing many ciphertexts and plaintext-ciphertext pairs.
- Question: Can the Dolev-Yao attacker do more to break the scheme than the adversary here?

Why is ECB not secure under the model?

- ECB is deterministic cipher scheme
- Identical blocks in plaintext are identical blocks in ciphertext
- The adversary just has to mark  $m_0$  and  $m_1$  with identical plaintext pairs in different positions.
- $b=0$  if identical ciphertext blocks in positions as in plaintext  $m_0$ , else  $b = 1$

Why is Counter Mode (CTR) secure under the model?

- CTR is non-deterministic, random initialization vector
- Be careful, counter value must not repeat

- Cryptographic algorithms are deterministic algorithms
- Naively using them leads to deterministic crypto schemes which are not secure under CPA.
- However, deterministic encryption schemes have special use cases where they can make sense, but typically not in the security of network communication!
- When generating a crypto scheme to realize a security property, we usually need to generate a non-deterministic scheme.

- [KL15] Jonathan Katz and Yehuda Lindell, Introduction to Modern Cryptography, 2nd edition, CRC Press, 2015