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Tutorials for Network Coding (IN3300)

Tutorial 1 – 2014/10/21

Problem 1 FEC with ARQ

Consider a simple wireless network consisting of two nodes s and t. Node s transmits packets of $l = 15\,808\,\text{bit}$ each. The channel has a bit error rate of $\epsilon = 10^{-4}$.

If a transmission of s is successfully received by t, an acknowledgement is triggered and sent back to s. We assume orthogonal scheduling, i. e, there are no additional losses due to collisions. Further we assume that acknowledgements do not get lost lost.

a)* Let X be a random variable that counts the number of bit errors in a given packet. Determine the probability for a successful transmission, i. e., $\Pr[X = 0]$.

b) Let T denote a random variable that counts the number of transmissions until a packet is acknowledged. Determine $\Pr[T = i]$ and $\Pr[T \leq i]$ in general and for $i = 7$.

c) Determine the expectation $\mathrm{E}[T]$, i. e., the average number of transmissions that are needed until successful reception.

To secure transmissions node s now employs a FEC code which maps source symbols of $k = 247\,\text{bit}$ to coded symbols of $n = 255\,\text{bit}$. The code is able to detect and correct a single bit-error in each coded symbol.

d) Determine the probability that a single block can be recovered at the receiver.

e) Let Z count the number of incorrect transmitted symbols. Determine the probability for a successful transmission during the first attempt for the whole packet if FEC is used.

Problem 2 Linear dependency of random vectors

Let $\boldsymbol{c} \in F_q^n[x]$ denote coding vectors which are drawn independently and uniformly distributed. Coding vectors are assembled to a coding matrix $\boldsymbol{C} = [\boldsymbol{c}_1 \ \ldots \ \boldsymbol{c}_m] \in F_q^{n \times m}[x]$ at the receiver. The receiver is able to decode if $\operatorname{rank} \boldsymbol{C} = n$. Let ρ_{mn}^k denote the probability that $\operatorname{rank} \boldsymbol{C} = k \leq n$ after receiving $m \geq k$ coding vectors.

a)* Determine the probability ρ_{1n}^1, i. e., the probability to draw a random vector $\boldsymbol{c} \neq \mathbf{0}$.

b) Determine the probability ρ_{2n}^2, i. e., two random vectors are linear independent.

c) Determine the probability ρ_{mn}^{m} for $m \leq n$.

d) Determine the probability ρ_{mn}^{n} for $m \geq n$.

Let X denote a random variable counting the number of vectors needed to span $F_q^n[x]$. The probability for $X < n$ is obviously 0. For $m > n$ the probability is given by $\rho_{m-1,n}^{n}$ and thus

$$\Pr\left[X < m\right] = \begin{cases} 0 & m \leq n, \\ \rho_{m-1,n}^{n} & m > n. \end{cases} \tag{1}$$

e)* Derive $\mathrm{E}[X]$ for $n = 32$ and $q \in \{2, 8, 16, 32\}$. As far as we know$\mathrm{E}[X]$ has no closed form. So simplify as much as possible and then use Matlab to determine numerical results.

Hint: $\mathrm{E}[X] = \sum_{m=1}^{\infty} \Pr\left[X \geq m\right]$.